XLVI. *Problems by* Edward Waring, *M A.* *and* Lucaſian *Profeſſor of Mathematics in the Univerſity of* Cambridge, *F. R. S.*

PRO.

Read April 21, 1763.

1. INvenire, quot radices impoſſibiles habet data biquadratica æquatio $x^4 + qx^2 - rx + s = 0$.

1^mo Sit $256 s^3 - 128 q^2 s^2 + \overline{144 r^2 q + 16 q^4} \times s - 27 r^4 - 4 r^2 q^3$ negativa quantitas, & duas & non plures impoſſibiles radices habet data æquatio.

2^do Sit affirmativa quantitas, & vel $-q$ vel $q^2 - 4s$ negativa quantitas, & datæ æquationis quatuor radices erunt impoſſibiles.

3^tio. Sit nihilo æqualis, & vel $-q$ vel $q^2 - 4s$ negativa quantitas, & datæ æquationis duæ inæquales radicis erunt impoſſibiles.

2. Invenire, quot radices impoſſibiles habet data æquatio $x^5 + q x^3 - r x^2 + s x - t = 0$.

1^mo Si ſigna terminorum æquationis $w^{10} + 10 q w^9$
$+ \overline{39 q^2 + 10 s} \times w^8 + \overline{80 q^3 + 50 q s + 25 r^2} \times w^7 +$
$\overline{95 q^4 + 124 q^2 s - 95 s^2 + 92 q r^2 + 200 r t} \times w^6 +$
$\overline{66 q^5 - 360 q s^2 + 196 q^3 s + 118 q^2 r^2 + 260 r^2 s + 625}$
$\overline{t^2 + 400 q r t} \times w^5 + \overline{25 q^6 + 40 s^3 - 53 r^4 + 52 q^3 r^2 -}$
$\overline{522 q^2 s^2 + 194 q^4 s + 708 q r^2 s + 240 q^2 r t + 1750}$
$\overline{q t^2 - 950 s r t} \times w^4 + \overline{4 q^7 + 106 q^5 s - 80 q s^3 - 308}$
$\overline{q^3 s^2 - 102 q r^4 - 7 q^4 r^2 + 570 r^2 s^2 + 612 q^2 r^2 s + 700}$
$\overline{r^2 t - 3750 t^2 s + 2500 t^2 q^2 + 80 r t q^3 - 2150 q r s t}$
$\times w^3 + 400 s^4 - 360 q^2 s^3 - 15 q^4 s^2 + 24 q^6 s - 8 q^5 r^2$
$- 45$

$-45q^2r^4-270r^4s+140r^2sq^3+960r^2s^2q+1875t^2r^2+1000trs^2-5000t^2qs+1750t^2q^3+40trq^4+600tr^3q-1650trsq^2\times w^2+36q^5s^2-224q^3s^2+320qs^4+4q^3r^4+27r^6-40r^2s^2+434r^2q^2s^2-24r^2sq^4-198r^4qs+5000t^2s^2-450tr^3s-6250t^3r+675t^2q^4-3750t^2q^2s+3000t^2r^2q+60tr^3q^2+200trs^2q-330trq^3s\times w+3125t^4-3750qrt^3+2000s^2q+2250r^2s-900sq^3+825r^2q^2+108q^5\times t^2-1600s^3r-560rq^2s^2-16r^3q^3+630r^3qs+72rsq^4-108r^5\times t+256s^5-128q^2s^4+144r^2qs^3+16q^4s^3-27r^4s^2-4r^2q^3s^2=0$. continuo mutentur de + in —; & — in +; nullas impossibiles radices habet data æquatio.

2[do]. Si signa terminorum æquationis haud continuo mutentur de + in — & — in +; duæ vel quatuor datæ æquationis radices erunt impossibiles, prout ultimus ejus terminus sit negativa vel affirmativa quantitas.

3[tio]. Si ultimus ejus terminus nihilo sit æqualis, & signa terminorum æquationis haud continuo mutentur de + in — & — in +; tum vel quatuor vel duæ radices datæ æquationis erunt impossibiles, prout duo & non plures ultimi datæ æquationis termini nihilo sint æquales, necne.

P R O.

Sint x, y, v, abscissa, ordinata & area datæ curvæ, & sit $y^n+\overline{a+bx}\times y^{n-1}+\overline{c+dx+ex^2}\times y^{n-2}+\overline{f+gx+hx^2+kx^3}\times y^{n-3}+$ &c. $=0$: invenire, utrum area (v) quadrari potest, necne.

Supponamus æquationem ad aream esse $v^n+\overline{A+Bx+Cx^2}\,v^{n-1}+\overline{D+Ex+Fx^2+Gx^3+Hx^4}\times$

$v^{n-2} + \overline{I + Kx + Lx^2 + Mx^3 + Nx^4 + Ox^5 + Px^6} \times v^{n-3} + \&c. = 0.$ & consequenter erit

$$\left.\begin{array}{l} nyv^{n-1} + \overline{n-1}\,\overline{A+Bx+Cx^2}\,yv^{n-2} + \overline{n-2} \times \overline{D \times Ex+Fx^2+Gx^3+Hx^4} \times yv^{n-3} + \&c. \\ \overline{B+2Cx}\, v^{n-1} + \overline{E+2Fx+3Gx^2+4Hx^3} \times v^{n-2} + \&c. \end{array}\right\} = 0.$$

Ex quibus æquationibus, si methodis notis exterminetur (v), habebimus æquationem, quæ exprimit relationem inter (x) & (y). Hujus autem æquationis coefficientes æquari debent coefficientibus datæ æquationis $y^n + \overline{a+bx}\,y^{n-1} + \overline{c+dx+ex^2} + y^{n-2} + \&c. = 0$; & si quantitates A, B, C, &c. exinde determinari possunt, curva quadratur, est enim $v^n + \overline{A+Bx+Cx^2} \times v^{n-1} + \overline{D+Ex+Fx^2+Gx^3+Hx^4} \times v^{n-2} + \&c. = 0$; aliter autem quadrari non potest.

Ex. Sit data æquatio $y^2 + x^2 - 1 = 0$, & supponamus æquationem ad aream $v^2 + D + Ex + Fx^2 + Gx^3 + Hx^4 = 0$; & erit $2vy + E + 2Fx + 3Gx^2 + 4Hx^3 = 0$, ita reducantur hæ duæ æquationes in unam, ut exterminatur (v), & resultat æquatio $y^2 + \frac{16H^2x^6 + 24HGx^5 + \overline{16HF+9G^2}\,x^4 + \overline{8EH+12FG}\,x^3 + \overline{6GE+4F^2}\,x^2 + 4FEx + E^2}{4 \times \overline{Hx^4+Gx^3+Fx^2+Ex+D}} = 0$; debet autem fractio $\frac{16H^2x^6 + 24HGx^5 + \overline{16HF+9G^2}\,x^4 + \overline{8EH+12FG}\,x^3 + \overline{6GE+4F^2}\,x^2 + 4FEx + E^2}{4 \times \overline{Hx^4+Gx^3+Fx^2+Ex+D}}$ esse $x^2 - 1$; & consequenter

$$4H = 16H^2$$
$$4G = 24HG$$
$$4F - 4H = 16HF + 9G^2$$
$$4E - 4G = 8HE + 12FG$$
$$4D - 4F = 6GE + 4F^2$$
$$-4E = 4FE$$
$$-4D = E^2$$

ſed e methodo communes diviſores inveniendi conſtat has æquationes inter ſe contradictorias eſſe, & conſequenter curvam haud generaliter eſſe quadrabilem.

THEO.

Sint x, y, v, abſciſſa & ordinatæ curvarum ABCD EFGHI &c. & $A\beta\gamma\delta\varepsilon$ &c. & ſit $y = px^n$, & $v =$

$$\frac{n}{2.3} pa^{n-1}x - \frac{n \times \overline{n-1} \times \overline{n-2}}{30 \times 2 \times 3} pa^{n-3}x^3 + \frac{n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4}}{42 \times 2 \times 3 \times 4 \times 5} pa^{n-5}x^5 - \frac{n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4} \times \overline{n-5} \times \overline{n-6}}{30 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7} pa^{n-7}x^7 + \frac{5n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4} \times \overline{n-5} \times \overline{n-6} \times \overline{n-7} \times \overline{n-8}}{66 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9} pa^{n-9}x^9 - \frac{691 \times n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4} \times \overline{n-5} \times \overline{n-6} \times \overline{n-7} \times \overline{n-8} \times \overline{n-9} \times \overline{n-10}}{2730 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11} pa^{n-11}$$

x^{11} + &c. cujus ultimus terminus debet eſſe x^{n-1} vel x^{n-2}, prout (n) eſt par vel impar numerus.

Sit $x = AP = a$, biſecetur AP in T in duas æquales partes, & ducatur linea $ET\delta$, & ſi AE, EM, AM, jungantur; erit triangulum AEM $=$ TP $\varepsilon\delta$ T areæ.

Deinde,

Deinde, bisecentur TP, AT in R and V, & ducantur RG, CVγ; & jungantur AC, CE, EG, GM; & erunt duo triangula ACE + EGM = VT $\delta\gamma$ V areæ.

Eodem modo, si partes AV, VT, TR, RP iterum bisecentur in W, U, S, Q, & ducantur lineæ BWβ, UD, SF, QH; & jungantur AB, BC, CD, DE, EF, FG, GH, HM; erunt quatuor triangula ABC + CDE + EFG + GHM = WV $\gamma\beta$ W areæ; & sic deinceps.

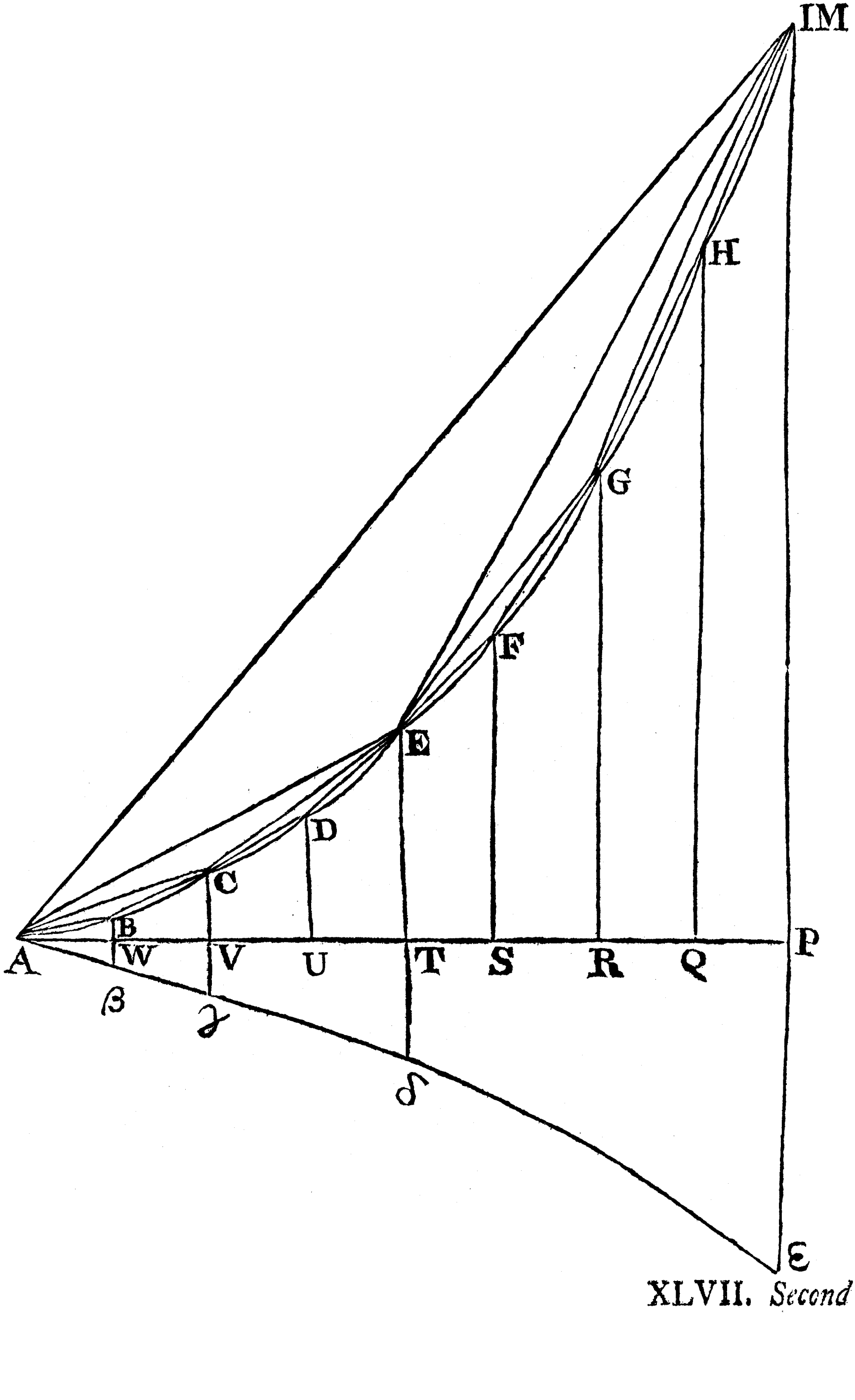
Cor. 1. Si curva ABC & M sit conica parabola, $(c, e)\ y = pix^2$, erit $v = \frac{1}{3}pax$; & A$\beta\gamma\delta$ &c. erit recta linea; & propositio eadem est cum notissimâ propositione Archimedis de quadraturâ parabolæ.

Cor. 2. Si $y = px^3$, erit $v = \frac{1}{2}pa^2x$, & A$\beta\gamma\delta$ &c. iterum recta linea.

Cor. 3. Datâ curvâ, cujus æquatio est $y = px^{2n}$, inveniri potest altera curva, cujus dimensiones sunt $(2n-1)$, in quâ summæ triangulorum ad singulas bisectiones erunt respectivè æquales summis triangulorum datæ curvæ.

His adjici potest, quod si loco bisectionis abscissa AP aliâ quâvis ratione in æquales partes dividatur, summæ triangulorum curvæ ABCD &c. ad singulas divisiones æquales erunt segmentis curvæ A$\beta\gamma\delta$ &c.

IM



XLVII. *Second*